Gravitational Leptogenesis and Its Signatures in CMB

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We study the phenomenologies in astrophysics and particularly in CMB associated with the gravitational leptogenesis. Our results show that future CMB polarization experiments, such as PLANCK and CMBpol will make a possible test on this class of model for leptogenesis.

The origin of the baryon number asymmetry in the universe remains a big puzzle in cosmology and particle physics. Sakharov's original proposal for a dynamical generation of the baryon asymmetry requires three ingredients [1]: (1) baryon number violation; (2) C and CP non-conservation; and (3) out of thermal equilibrium. All of the three conditions above in the minimal standard model (SM) of the particle physics can be realized, however, quantitatively the baryon asymmetry generated during the electroweak phase transition is too small to account for the value observed [2]: $n_B/s \sim 10^{-10}$. In the SM, the CP violation is not enough and the first order phase transition is too weak. Thus far in the literature many models involving new physics, such as multi-Higgs models, left-right symmetric models, supersymmetric models and models with extra dimension have been proposed for baryogenesis.

Theoretically there exist many possible ways to go beyond the SM. Experimentally a strong evidence for new physics beyond the SM comes from the establishment on the atmospheric and the solar neutrino oscillation with an additional support from the reactor antineutrino, which demonstrates that the neutrinos have masses and mix with each other. Within the content of the particles in the SM the neutrino masses and mixing can be described by a dimension-5 operator

$$\mathcal{L}_{\not{L}} = \frac{2}{f} l_L l_L \phi \phi + \text{H.c.} , \qquad (1)$$

where f is the scale of new physics beyond the Standard Model, l_L and ϕ are the left-handed lepton and the Higgs doublets respectively. When the Higgs field gets a vacuum expectation value $< \phi > \sim v$, the left-handed neutrino receives a Majorana mass $m_\nu \sim \frac{v^2}{f}$.

With the operator in (1) the baryon minus lepton $(B-L)$ number is violated. It would be very economical if the leptogenesis [3] happens in this minimal extension of the SM. However the Sakharov's third condition can not be realized. In the traditional version of the leptogenesis the heavy right-handed neutrinos are introduced and their non-thermal equilibrium decays, coupled with the electroweak sphaleron process, generate the required baryon number asymmetry. In general at least two types of the right-handed neutrinos are needed for a successful leptogenesis.

Note that the Sakharov's third condition applies for models where the CPT is conserved. If the CPT is violated the baryogenesis or leptogenesis could happen in thermal equilibrium [1], such as the spontaneous baryogenesis [6] and the gravitational baryogenesis[7]. In the original version of the spontaneous baryogenesis by Cohen and Kaplan [6] it requires an extra scalar field beyond the SM and its derivative coupling to the baryon current. In models of gravitational baryogenesis[7] [8] [9] the scalar field in [6] is replaced by a function of the Ricci scalar,

$$\mathcal{L}_{int} = c \, \partial_\mu f(R) J^\mu , \qquad (2)$$

where J^μ is a vector current made of the particles in the standard model, $f(R)$ a dimensionless function of the Ricci scalar R and c is the coupling constant characterizing the strength of this interaction. In Ref.[7] $f(R) = R/M^2$ is taken, however the Einstein equation, $R = 8\pi G T^\mu_\mu = 8\pi G(1 - 3w)\rho$, tells us that $\dot{f}(R) = 0$ in the radiation dominated epoch. In Ref.[8] we have proposed a model of gravitational leptogenesis

$$\frac{n_{B-L}}{s} \sim c \frac{\dot{f}(R)}{T} , \qquad (3)$$

where $f(R) \sim \ln R$, so the term $\partial_\mu f(R) \sim \partial_\mu R/R$ does not vanish during the radiation dominated epoch and the observed baryon number asymmetry can be generated naturally via leptogenesis. Since in this scenario of baryogenesis, no extra degrees of freedom beyond the SM and classical gravity are introduced we call it the minimal model of baryogenesis.

The key ingredient in this type of leptogenesis is the cosmological CPT violation caused by the non-vanishing $\dot{f}(R)$. It would be very interesting to ask if this type of CPT violation could be tested in the experiments. Unfortunately in the laboratory the predicted CPT violation as shown in [8] is much below the current experimental sensitivity. In this paper we will show that the phenomenon associated with the CPT violation can be tested in the future cosmic microwave background

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[1] For recent relevant works see, e.g., [4, 5].

(CMB) polarization experiments, such as PLANCK[32] and CMBpol[33]. For this purpose, we take specifically the J^μ in (2) to be the Chern-Simons term of the electromagnetic field,

$$\mathcal{L}_{int} = -\frac{1}{2}\delta \ \partial_\mu f(R) K^\mu \ , \qquad (4)$$

where $K^\mu = A_\nu \widetilde{F}^{\mu\nu}$, $F_{\mu\nu}$ is the electromagnetic field strength tensor, and $\widetilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is its dual. In (4) δ is a constant characterizing the strength of this type of interaction and will be calculable if the underlying fundamental theory is known. In the framework of effective theory, we expect it by a naive dimensional analysis to be suppressed by a factor of $\frac{e^2}{4\pi^2}$ in comparison with the constant c in (3). The factor $-\frac{1}{2}$ in (4) is introduced for the convenience of the discussions.

The term in (4) is gauge invariant and parity-odd. And it can lead to the rotation of electromagnetic waves when propagating over cosmological distances [10]. This effect is known as "cosmological birefringence". By observing the change $\Delta\chi$ in position angle χ of the plane of polarized radiation from distant radio galaxies and quasars at a redshift z due to the phase shift between the modes of opposite helicity, cosmological birefringence is directly observable and has been used to constrain the amplitude of Lorentz and parity-violating modifications to electrodynamics [10, 11]. Refs.[12, 13, 14, 15, 16, 17, 18, 19] have considered some effects of the cosmological birefringence in CMB.

We start our discussion with an examination on the experimental limits from quasars on the coupling constant δ in Eq. (4). We will show that, for $f(R) \sim \ln R$, the experimental constraints on δ are not much stringent, e.g., $|\delta| \lesssim \mathcal{O}(10^{-1})$. Then we will study the effects of our model on the polarization of CMB. Our results show that for $|\delta| \gtrsim \mathcal{O}(10^{-7})$, the interaction in Eq. (4) will give rise to effects observable in the future CMB polarization experiments.

With the interaction in Eq. (4), the Lagrangian for the electromagnetic field in the absence of source is

$$\mathcal{L}_{em} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{int} \ . \qquad (5)$$

The equations of motion for the electromagnetic field are

$$\nabla_\mu F^{\mu\nu} = \delta \ \partial_\mu f \widetilde{F}^{\mu\nu} \ , \qquad (6)$$

and

$$\nabla_\mu \widetilde{F}^{\mu\nu} = 0 \ , \qquad (7)$$

where ∇_μ denotes the covariant derivative with the metric $g_{\mu\nu}$. In the spatially flat FRW cosmology, the metric can be written as

$$ds^2 = a^2(\eta)(d\eta^2 - \delta_{ij}dx^i dx^j) \ , \qquad (8)$$

where η is the conformal time which is related to the cosmic time by $d\eta = dt/a$. With the conventions, $A^\mu = (A^0, \mathbf{A})$ and

$$\mathbf{E} = -\nabla \mathbf{A^0} - \frac{\partial \mathbf{A}}{\partial \eta} \ ,$$
$$\mathbf{B} = \nabla \times \mathbf{A} \ , \qquad (9)$$

we can write the electromagnetic field strength tensor in terms of $\mathbf{E}$ and $\mathbf{B}$:

$$F^{\mu\nu} = a^{-2}\begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix} \ . \qquad (10)$$

In Eq. (9), ∇ and $\nabla\times$ represent the usual gradient and curl operators in the Cartesian three dimensional space. The dual tensor $\widetilde{F}^{\mu\nu}$ can be obtained from $F^{\mu\nu}$ by replacing $\mathbf{E}$ and $\mathbf{B}$ with $\mathbf{B}$ and $-\mathbf{E}$ respectively. In terms of the notations given by Carroll and Field [20]:

$$\mathbf{B}(\vec{\mathbf{x}}, \eta) = e^{-i\mathbf{k}\cdot\mathbf{x}}\mathbf{B}(\eta) \ ,$$
$$F_\pm = a^2 B_\pm(\eta) = a^2(B_y \pm iB_z) \ , \qquad (11)$$

we have the equation of motion for a given mode $\mathbf{k}$,

$$F_\pm'' + (k^2 \pm \delta k f')F_\pm = 0 \ , \qquad (12)$$

where the prime represents the derivative with respect to η and k is the modulus of $\mathbf{k}$. In the equation above, we have assumed the wave vector $\mathbf{k}$ is along the x axis, and $+$ and $-$ denote the right- and left-handed circular polarization modes respectively. The non-vanishing $f'(R)$ induces some difference between the dispersion relations for the modes with different handedness. This will rotate the direction of the polarization of light from distant sources. For a source at a redshift z, the rotation angle is

$$\Delta\chi = \frac{1}{2}\delta \ \Delta f(R) \ , \qquad (13)$$

where Δf is the change in f between the redshift z and today, i.e., $\Delta f = f|_z - f|_{z=0}$.

For $f(R) \sim \ln R$ and in the spatially flat ΛCDM model, we get

$$\Delta f = \ln\left(\frac{\Omega_{m0}(1+z)^3 + 4\Omega_{\Lambda 0}}{\Omega_{m0} + 4\Omega_{\Lambda 0}}\right) \ . \qquad (14)$$

The subscript 0 denotes today's value. The underlying model parameters we set below are from Ref. [21]:

$$(\tau, \Omega_\Lambda, \Omega_d h^2, \Omega_b h^2, A_S, n_S) = (0.17, 0.72, 0.12, 0.024, 0.89, 1) \ . \qquad (15)$$

Despite the fact argued in the literature [22, 23, 24] whether cosmic birefringence has been detected through distant radio galaxies and quasars, using the data given

by Leahy [23] we can get a conservative(see also [24]) limit on the coefficient δ in our model. As an illustrative effect, in Fig.1 we show three resulting $\Delta\chi - z$ effects for different values of δ. We can see from Fig.1 that the single source 3C9 at $z = 2.012$, which reads $\Delta\chi = 2° \pm 3°$ and is consistent with the detailed analysis of [24], restricts stringently the amplitude of δ to be no more than the order of 0.1. Furthermore, in order to have an estimation on $|\delta|$, we fit our model to the data with $\Omega_m = 0.3 \pm 0.08$, which is around the 2σ limit of the 6-parameter global fit. The corresponding $2-$ dimensional and $1-$ dimensional plots with 1 and 2σ $C.L.$ are delineated in Fig.2. We get at 1σ limit $\delta = 0.03 \pm 0.07$ from the $1-$ dimensional case.
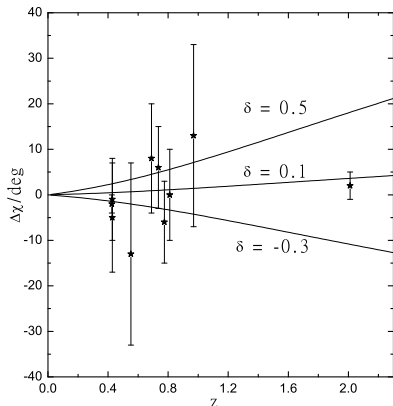

FIG. 1: Different effects for choosing three sets of δ of our model in light of the data from distant radio sources from [23].
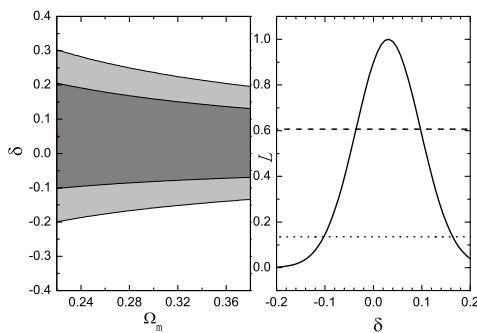

FIG. 2: $2-$ dimensional and $1-$ dimensional constraints on δ in light of the data from distant radio sources from [23], with 1 and 2σ $C.L.$ shown respectively.

Now we consider the effects of the interaction (4) in CMB. As is well known the CMB has become the most remarkable tool for probing the distant past and present status of our universe. In particular, DASI [25] has marked the first detection of G-mode polarization and TG correlation of CMB. The first year WMAP measurement [26] has been up to date the most powerful observation of cosmology, both in deriving very tight constraints on cosmological parameters and revealing new signatures of our universe. The polarization of CMB helps particularly to constrain the reionization depth [27, 28] and verify the existence of early cosmological inflation [29] in high precision surveys like WMAP. Cosmological birefringence rotates the gradient type polarization fields into the curl type during the propagations of the CMB photons from the last scattering surface to the observer, leading to distinctive effects in the CMB polarization surveys [12, 30, 31] which leaves hope for being directly detected in high precision CMB measurements like PLANCK [32] and CMBpol [33].

The CMB polarization can be described by two Stokes parameters: Q and U, which can be spherically expanded to get a gradient (G) and a curl (C) component [34]. If the temperature/polarization distribution does not violate parity, one gets vanished CMB TC and GC due to the intrinsic properties of the tensor spherical harmonics. In the case of P violation effect which leads to cosmological birefringence, the polarization vector of each photon is rotated by an angle $\Delta\chi$ everywhere and one would get nonzero TC and GC correlations with

$$C_l^{TC} = C_l^{TG} \sin 2\Delta\chi \ , C_l^{GC} = \frac{1}{2}(C_l^{GG} - C_l^{CC}) \sin 4\Delta\chi \ , \tag{16}$$

even though they vanish at the surface of the last scattering.

To model the CMB polarization experiments we follow the notations of Refs. [17, 18, 19]. For a full sky pixelized map with a Gaussian beam with full width at half maximum θ_{FWHM}, we assume uncorrelated errors of pixels with uniform variance σ_T^2 and σ_P^2 respectively for temperature and polarization measurements. Assuming negligible gravitational wave contributions, we show in Fig. 3 the smallest δ detectable at 1σ level using CMB TC signature only in the left panel and using CMB GC signature in the right panel with given θ_{FWHM} and σ_T. Fig. 3 shows CMB GC polarization, which has not been studied quantitatively before, serves more efficient than TC measurements due to different intrinsic properties of the curl and gradient polarizations. PLANCK can detect a CMB GC signature for $|\delta| \sim 10^{-5}$ while the $|\delta|$ as small as $\sim 10^{-7}$ is detectable in CMBpol experiment. We point out here that our result applies to a general class of cosmological birefringence and CMB GC signature in CMBpol can detect $\Delta\chi > 0.0001°$.

Before concluding we point out that this study about the sensitivity on the coefficient in the interaction (4) from the future CMB measurements has an interesting implication on the minimal model of the baryogenesis
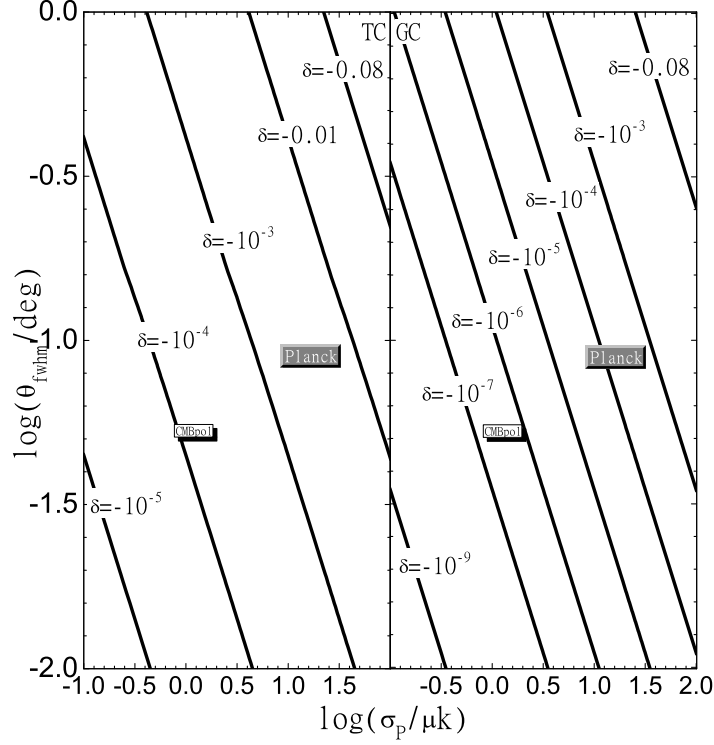
FIG. 3: The smallest δ detectable at 1σ level using CMB TC and GC signature with given experimental parameters. The filled and hatched areas illustrate the mission characteristics of PLANCK and CMBpol respectively.

[8]. Given Eq. (3) and $f(R) \sim \ln R$, the final $B - L$ number asymmetry generated is

$$\left.\frac{n_{(B-L)}}{s}\right|_{T_D} \simeq 0.1 c \frac{T_D}{m_{pl}} \ , \qquad (17)$$

where T_D is the decoupling temperature determined by the $B - L$ violating interaction in (1).

In the early universe the $B - L$ violating rate induced by the interaction in (1) is [35]

$$\Gamma_{\not L} \sim 0.04 \ \frac{T^3}{f^2} \ . \qquad (18)$$

Since $\Gamma_{\not L}$ is proportional to T^3, for a given f, $B - L$ violation will be more efficient at high temperature than at low temperature. Requiring this rate to be larger than the universe expansion rate $H \sim 1.66 g_*^{1/2} T^2 / m_{pl}$ until the temperature T_D, we obtain a T_D-dependent lower limit on the neutrino mass:

$$\sum_i m_i^2 = (0.2 \ \text{eV} (\frac{10^{12} \ \text{GeV}}{T_D})^{1/2})^2 \ . \qquad (19)$$

The experimental bounds on the neutrino masses come from the neutrino oscillation experiments and the cosmological tests. The atmospheric and solar neutrino oscillation experiments give[36, 37]:

$$\Delta m_{atm}^2 = (2.6 \pm 0.4) \times 10^{-3} \text{eV}^2 \ , \qquad (20)$$

$$\Delta m_{sol}^2 \simeq (7.1^{+1.2}_{-0.6}) \times 10^{-5} \text{eV}^2 \ . \qquad (21)$$

The cosmological tests provide the limits on the sum of the three neutrino masses, $\Sigma \equiv \sum_i m_i$. The analysis of WMAP [28] and SDSS [21] show the constraints: $\Sigma < 0.69$ eV and $\Sigma < 1.7$ eV respectively.

For the case of normal hierarchy neutrino masses, $m_3 \gg m_2$, m_1, one has

$$m_3^2 - m_2^2 = \Delta m_{atm}^2 \ , \quad m_2^2 - m_1^2 = \Delta m_{sol}^2 \ , \qquad (22)$$

and

$$\sum_i m_i^2 \simeq m_3^2 \gtrsim \Delta m_{atm}^2. \qquad (23)$$

We can see from Eq. (19) that this requires the decoupling temperature $T_D \lesssim 1.5 \times 10^{13}$ GeV. For neutrino

masses with inverted hierarchy, $m_3 \sim m_2 \gg m_1$, we get

$$m_3^2 - m_2^2 = \Delta m_{sol}^2 , \quad m_2^2 - m_1^2 = \Delta m_{atm}^2 , \quad (24)$$

and

$$\sum_i m_i^2 \simeq 2m_3^2 \gtrsim 2\Delta m_{atm}^2. \quad (25)$$

It constrains the decoupling temperature as $T_D \lesssim 7.7 \times 10^{12}$ GeV. If three neutrino masses are approximately degenerated, i.e. , $m_1 \sim m_2 \sim m_3 \sim \bar{m}$, one has $\Sigma = 3\bar{m}$ and $\sum_i m_i^2 \simeq \Sigma^2/3$. In this case, the WMAP and SDSS data require T_D to be larger than 2.5×10^{11} GeV and 4.2×10^{10} GeV respectively. So, for a rather conservative estimate, we consider T_D in the range of 10^{10}GeV $\lesssim T_D \lesssim 10^{13}$GeV. Combined with Eq. (17) for a successful leptogenesis this results in a constraint on the coupling constant c: $|c| \gtrsim 10^{-3}$. As argued above the dimensional analysis indicates $\delta \sim \frac{e^2}{4\pi^2} c$. The upper limit on $|c|$ for a successful baryogenesis implies that $|\delta| \gtrsim 10^{-6}$ which as we show above lies in the range sensitive to the future CMB measurements.

In summary, we have in this paper studied the phenomenon associated with the gravitational baryo(lepto)genesis in CMB. In this type of models for baryo(lepto)genesis no extra degrees of freedom beyond the standard model of particle physics and classical gravity are introduced, however the effective operators involving the derivative couplings between the standard model particles and the gravity play an important role in the generation of baryon number asymmetry. These operators give rise to CPT violation during the evolution of the Universe. In this paper we have considered explicitly the effects of the effective operator (4) in CMB and showed that the future CMBpol experiments can test this type of effective interaction for $|\delta|$ as small as 10^{-7}. Our results have an interesting implication in gravitational baryogenesis and show a possible way to test this type of baryogenesis in the future CMB polarization experiments, such as PLANCK and CMBpol.

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